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## The Continuous Combinatorial Auction Architecture

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# The Continuous Combinatorial Auction Architecture

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The paper reviews the prominent features of a continuous, combinatorial auction. The Section 1 is an introduction that provides some background research and overview. The continuous combinatorial auction evolved in response to experience with combinatorial auctions based on bidding rounds. The background material helps with the contrasts related to auction architectures. Section 2 consists of a few background definitions. Section 3 develops the rules and procedures, which are central features of the mechanism. Section 4 outlines important operational features such as information and query functions. As should become clear, the continuous combinatorial auction does not compute or even use prices per item. The fashioning of bids on packages of items is not guided by a sum of the prices of items in the package. Section 5 contains illustrations of the interfaces, which summarize how the mechanism operates with human participants. The section consists of a series of screen shots that illustrate how the system looks from the point of view of a bidder. Section 6 addresses performance in experiments and in the field.

## 1. Introduction and Background

The history of the continuous combinatorial auction marks the evolution of the mechanism. The concept of a combinatorial auction is due to Rassenti, Smith, and Bulfin (1982) who were motivated by the use of simultaneous ascending price auctions to allocate landing rights (Grether, Isaac, and Plott, (1979 and subsequently published as 1989)). The ideas were generalized by Banks, Ledyard, and Porter (1989) to include the concept of a “standby queue” which serves a function similar to non-winning bids in the current system.

The first example of a continuous combinatorial auction is found at Brewer and Plott (1996). They demonstrate that representations in terms of binary confects of packages afford both the flexibility for widespread application and the computational speed required to support the auction. This early mechanism depended heavily on the existence of a fixed set of packages on which bids could be placed. The computer could quickly compute non-intersecting packages that maximized the value of the sale and permitted the auction to proceed as a type of continuous, simultaneous, ascending price auction. The packages played the role of items on which bids were placed. The non- intersecting packages that produced the most revenue from the auction were declared the leading bids at each instant of time. That first mechanism was followed by slight generalization to a procurement problem in which the buyer organized sellers to minimize procurement cost and sellers could offer endogenous packages. The organization was a simultaneous, decreasing price auction.

In the 1990s, the FCC was considering the adoption of a combinatorial auction as a replacement for the simultaneous, rounds-based, ascending price auction that the FCC had used to auction parts of the electromagnetic spectrum. The initial research focused on a hybrid process that consisted of rounds followed by a continuous phase. (See the report of Charles R. Plott, FCC Why River Conference, May 5-7, 2000). Experiments with the hybrid revealed that, that most of the adjustment and efficiency came the continuous phase. That discovery led to the study of combinatorial auctions that operated only in continuous time. Of course, the architectures of the continuous combinatorial auctions and the traditional auctions based on rounds have some similarities but they have many differences that required exploration.

A difference between the continuous auction and rounds auctions is the role and form of activity and eligibility requirements. The continuous auction uses neither, and the rounds based auctions use both. In the rounds auction, “eligibility” is a limitation on the packages on which bids can be placed and activity requirements dictate a reduction in eligibility if bidding activity is not adequate. It is a type of “use it or lose it” condition. By contrast, the continuous auction is based on special ending clocks that play an incentive role similar to activity and eligibility requirements but are much different in substance and performance.

## 2. Items and bids

Items.  $n$  items are for sale at the auction indexed  $Y = \{1, 2, \dots, n\}$ ;  
Let  $S \in \{0, 1\}^n$  be a combination, set, or package of items.

Individuals.  $m$  individuals participate in bids  $M = \{1, 2, \dots, m\}$ . Each bidder has an Identification Number known only to the bidder. It is possible to give a bidder several ID numbers if the bidder wants them.

Bids. A bid is a price and a package of items of the form  $b_j^i(S)$  where  $i$  is the index of the individual submitting the bid,  $j$  is the bid number as recorded in the system and  $S \subseteq Y$  is a package of items.

Let  $b_q^i(S_q)$  be bid number  $q$ , where  $i$  is the bidder and  $S_q \in \{0, 1\}^n$ . That is, the  $q^{\text{th}}$  bid was placed by  $i$ , for a dollar amount  $b_q^i$  for a package of items  $S_q$ .

Bid Properties. (i) Bids are submitted under “all or none” conditions. Either the entire package is accepted as a provisional winner or none of it is accepted. (ii) Multiple bids can be tendered. (iii) All bids remain in the system and can be selected as provisional winners unless cancelled.

(iv) Provisional winning bids cannot be cancelled.

## 3. Rules and Procedures

Provisional Winners. After each bid is submitted, the system publishes the set of provisionally winning bids. The provisional winners are bids in the set of bids that would

maximize the value of the sale if the auction concluded at that moment subject to the condition that no item is contained in more than one provisionally winning bid.

$B$  = all bids submitted and not cancelled.

$x_q \in \{0,1\}$  indicates whether or not the  $q^{\text{th}}$  bid was accepted as a provisional winner of the auction.

$W$  = Provisionally winning bids. Provisional winners are the subset of all bids,  $B$ , that maximize the value of the sale subject to the fact that no item is contained in more than one provisional winning bid.

That is, provisional winning bids are  $q: x_q = \text{argmax } R$ .

$R = \text{Max } \sum x_q b_q^i$  Subject to  $\sum x_q S_q \leq (1,1,1,\dots,1)$   
 $x_q : q \in B$

Non provisional winning bids. The bids that are not provisional winners remain in the system and play an important role. Notice that the computation of the provisional winners includes an examination of all bids in the system. Thus, a new bid can be partnered with a non provisionally winning bid such that an existing package is broken and the new bid and the partnering bid become provisional winners. The implication is that non winning bids exist in the system as potential partners or as the pieces of a complex coalition that can be assembled to replace large package bids as provisional winners. By placing a non-winning bid, the bidder is revealing a willingness to pay for a package, which, theoretically, could be a maximum willingness to pay in the absence of mistakes or conspiracies.

Notice that this is a complex calculation that could require an examination of all families of subsets of bids as candidates for provisionally winning. Obviously, this can be computationally challenging. Given the sizes of existing auctions and tests, it has not presented a problem.

Increment requirements. The increment requirement represents a major departure for standard auctions. When bids are on a single unit, increment requirements state that new bids must be some fixed increment above the price of the currently winning bids on the item alone. Thus, the bidding is progressively upward. The increment requirement of the continuous combinatorial auction when the bid is a package of items is much different. In order to be submitted, a bid need not be high enough to become a provisional winner. Furthermore, the increment requirement is not based on the sum of implicit prices computed for individual items.

The function of the increment rule as is the case with all increment rules, is to encourage bids to move the system to an equilibrium and to do so at a fast pace. Without special rules regarding increments, the system could be filled with bids that are dominated by existing bids. Let  $v(S)$  be the maximum value for which the set  $S$  could be sold given the bids in the system. This value is determined by computing the winning bids from all bids submitted given that the sale of items was restricted to the set  $S$ . Let  $k$  be the (constant)

increment required for bidding on a single item. For a bid  $b(S)$  to meet the increment, it must meet the condition  $b(S) \geq v(S) + k |S|$ .

Stopping Rules and Warning Lights. Two clocks are used: a new bid clock and a new provisional winner clock. The new bid clock resets with each new bid and starts a countdown. Typically, the reset time is from three to five minutes. The time can be shortened as the auction progresses. The new winner clock resets each time a bid is placed that determines a new pattern of provisional winners. Typically, the reset time is between ten and fifteen minutes. Like the new bid clock the time can be shortened as the auction proceeds.

The auction ends if either clock reaches zero. Basically, the new bid clock forces a flow of bids, similar to offers in a negotiation and does so under the threat of the auction ending. The new winner clock forces concessions of sufficient magnitude to advance the value of the sale. In essence, the new bid clock says “You must make an offer. You must make an offer.” The new winner clock says “You must make an offer sufficiently to get a deal done or the auction ends anyway.” Thus, the ultimatum feature of game theory is operational in both clocks.

If either clock gets within one minute of zero, red “railroad lights” begin to blink on the screen. In essence, the system is constantly pressing for revenue gains by using the threat of ending the auction. While bidders who are not provisional winners do face a dominant strategy of bidding as the clock counts down, bid need not be large and there is no advantage to waiting until the last moment to bid. Last moment bids just give competitors more time.

Special bids, Robots and Either/or. Bidders are able to place an either/or bid. If one bid becomes a provisional winner, the other cannot. This feature allows expression of indifference across sets.

Robots are available for bidding on single items. The bidder can instruct the robot to bid no more than a stated amount that the bidder can change at any time. The robot will place a bid at the minimum increment any time the bid in question is no longer a provisional winner. Because bidding on sets requires fashioning bids, the robots are not available for bidding on sets.

#### 4. Information and Query

Bidders use the information and query functions to fashion bids. In particular, the bidders are given information needed about an entire package as opposed to some measure of implicit prices of the items in the package. In contrast to standard rounds based auctions, the system does not compute a measure of individual prices, the sum of which will indicate whether or not the package will be a provisional winner. Of course, since the system does not compute item prices, substantial information must be made

available to bidders in some other form. Some of this can best be understood by a study of the interfaces presented in Section 5 but a key list is included here.

As will become clear, the computations are complex, basically NP complete. However, unlike many auction architectures, the system need not compute prices or temporary equilibria based on preferences over all items, submitted by all bidders at the same time. Instead, the system responds to a single addition to the bids that exist in the system. Conventions exist to respond to computations that are taking too long for fast progression of the auction.

Provisional Winners. A table is published that contains the provisional winner of each item, whether the provisionally winning bid is a package bid or a single, the highest bid placed on the item as a single, and the highlights of the bids of the bidder who is doing the bidding.

The provisionally winning table is updated with each new bid. New provisionally winning bids are accompanied by a small red dot that disappears in a few seconds. All items in a new winning package appear with the red dot.

New non-winning bids are shown as a small black dot on the provisionally winning table. The dots also disappear after a few seconds. These black dots signal the possibility that a bidder wants part of a package but cannot bid enough to become a provisional winner and seeks partners to bid on the rest of the package from which the bidder wants a portion. The black dots carry information that serves to coordinate bidding by coalitions of small bidders who want to break up a large package bid.

All Bids. A page of all bids in the system is published. It includes the bid number, the bidder ID, the items in the package, the time of submission, and the amount of the bid.

Query. The query system and related functionality serve to replace the role of prices in the fashioning of bids. Important queries can be exercised at the time the bidder formulates a bid but before submission. When potential package is selected for a potential bid, the bidder is immediately shown both the minimum amount that can be bid as dictated by the increment requirements and the minimum amount it would take for the item to become a provisional winner. These two operations serve as tools to help bidders explore how to fashion bids in relation to the bids in the system. By adding or removing items from a package, the query can be used to determine the marginal cost of adding items to a package. By removing a single item from a package, the bidding required to become a provisional winner can be significantly reduced.

Show as Winning. After a bid is fashioned but before it is submitted, the bidder can choose this option to display the pattern of winning and non winning bids that will be the consequence of the submission. It will show all new provisional winners, all bids that were provisional winners and remain as such, all bids that were provisional winners and now are not, and all bids that were not provisional winners and would be provisional

winners if the bid was submitted. This allows bidders to search more efficiently for partners and avoid adding items to a package that would be too costly.

## 5. Interfaces

The interfaces presented here reflect what we have learned about what bidders want to know, aided by strategic considerations from game theory. When observing individual behavior in experiments, we follow the principle that the individual is an optimizer subject to perception of conditions and options available. Behavior that is not consistent with the incentives that we know exist are viewed as mistakes or misperceptions that the properly designed interfaces should prevent.

The interfaces produced in the illustrations below reflect the experience gained from experiments. The best way to explain interfaces seems to be to simply show them. The next seven pages are screen shots and explanations of the major functions and how they relate to bidder decisions. The illustrations begin with the home screen and provide a map to the other screens.

### Illustration 1. Bids, Provisional Winners, Clocks

The screenshot shows an auction interface with several key components highlighted by red circles and arrows. The interface is titled 'Auction - Microsoft Internet Explorer' and includes a 'New User?' link. A 'Watch List' is visible on the right side. The main area displays a table of 'Provisional Winners' and a 'Watch List' table. The 'Provisional Winners' table has columns for 'Item', 'Bid', 'Price', and 'Status'. The 'Watch List' table has columns for 'Item', 'Bid', 'Price', and 'Status'. The 'Provisional Winners' table shows items 101 through 208, with item 101 having a bid of 1.0000 and a price of 1.0000. The 'Watch List' table shows items 101 through 208, with item 101 having a bid of 1.0000 and a price of 1.0000. The interface also includes a 'Fashion Your Offer' section on the left, a 'Real Time or Paused' status indicator, and a 'View the Offers (Provisional Winners, Complete Offer List)' link on the right. A 'Watch the Clocks' link is also present. The interface is designed to provide a clear view of the current state of the auction and allow bidders to make informed decisions.

Watch the Clocks

Fashion Your Offer

Real Time or Paused

View the Offers  
(Provisional  
Winners,  
Complete Offer  
List)

## Illustration 2. Fashion and Submit an Offer

**1.** Make sure you are on one of these

**2.** Select the Item(s)

**3.** Add selected to your offer

**4.** Inspect selected before proceeding

**5.** Choose price

**6.** Short cut to price selection:  
a. Query for amount, b. choose minimum or winning

**7.** **Submit Offer**

run number			
101	102	103	104
105	106	107	108
109	110	111	112
113	114	115	116
117	118	119	120
121	122	123	124
125	126	127	128
129	130	131	132
133	134	135	136
137	138	139	140
141	142	143	144
145	146	147	201



Illustration 3. View the Offers (Provisional Winners)

run#, PIC making offer

1. Make sure you are here

**Purple**  
Provisional Winner of a single

**Blue PIC**  
Provisional Winner of a package offer

Black price  
high single offer (not winning due to a package offer)

**Gold** means you are the Provisional Winner

Red dot is a new Provisional Winner

Black dot is new non-winning offer

Provisional Winners			Watch List			Complete offer list					
run#	PIC	\$Offer	run#	PIC	\$Offer	run#	PIC	\$Offer	run#	PIC	\$Offer
101	12	17850	102	85	15550	103	12	15050	104	12	15150
105	12	15100	106	89	15550	107	12	15000	108	78	15150
109	78	15100	110	78	15000	111	**	15000	112	78	15000
113	12	15000	114	85	15150	115	**	15000	116	**	15000
117	90	15000	118	85	14950	119	12	15000	120	**	15000
121	12	15000	122	**	15000	123	12	15100	124	12	15000
125	12	14950	126	**	15000	127	85	15000	128	**	15000
129	**	15000	130	**	15000	131	**	15000	132	**	15000
133	**	15000	134	**	15000	135	**	15000	136	**	15000
137	13	15000	138	**	15000	139	**	15000	140	**	15000
141	**	15000	142	**	15000	143	**	15000	144	**	15000
145	**	15000	146	**	15000	147	**	15000	201	**	14500
202	78	14450	203	78	14450	204	**	14500	205	**	14500
206	**	14500	207	**	14500	208	**	14500	209	**	14500

# Illustration 4. Ending the Auction

Watch the clocks

Each new bid resets the new bid clock to three minutes (unless otherwise announced).

Each new Provisional Winner resets the new winner clock to ten minutes (unless otherwise announced).

The auction ends when either the new bid clock or the new winner clock counts down to zero. Time on both clocks means that the auction is still open for bidding on all items. When the auction ends the Provisional Winners become the Auction Winners.

PIC: .

Provision

run#	PI
101	3
105	3
109	7
113	4
117	3
121	3
125	2
129	2
133	*
137	1
141	1
145	*
202	7

run number

## Illustration 5. Offer Management ([Offer Modification](#))

The screenshot shows the 'Offer Submission Form' with the following fields and controls:

- new bid:** 00:00
- new winner:** 00:00
- Price:** 48300
- Bid #:** 1266
- Items:** 101 103 106
- Buttons:** min, win, Show Bid as Winning, remove item, Clear All, Show Intersecting Bids, Submit Offer, Send Query.
- Status:** paused

Annotations and steps:

- 1. Make sure you are here**: Points to the 'new bid' and 'new winner' fields.
- 2. Cancel a non Provisional Winning offer:**
  - Select the offer
  - Cancel Selected Bids
- 3.1 Modify an offer: Select the offer**: Points to the 'Submit Offer' button.
- 3.2 Modify an offer:**
  - Select a new price or c. Change package
  - d. Submit Offer
- 4. Return to [Offer Submission Form](#)**: Points to the 'Submit Offer' button.

**Offer Modification Form**

bid#	price	items	modify
1279	15000	127	<input type="radio"/>
1278	15100	123	<input type="radio"/>
1267	15100	105	<input type="radio"/>
1266	45800	102 114 118	<input checked="" type="radio"/>
1265	48300	101 103 106	<input type="radio"/>
<b>total</b>	<b>60800</b>		

**Buttons:** Cancel Selected Bids, Clear

**Annotations:**

- Total dollars for your provisional winners**: Points to the 'total' row in the table.
- 1267** with a checkmark: Points to the '1267' bid in the table.

## Illustration 6. View the Offers (**Complete Offer List**)

1. Make sure you are here

2. Return to [Provisional Winners](#) screen

[Complete Offer List](#)

[Provisional Winners](#)  
[Watch List](#)

PIC of bidder making the offer,  
Bid number, Offer price,  
Items in offer,  
Time entered

PIC	bid#	price	items	time entered
86	1280	15000	141	Sat Jun 22 08:07:35 2002
85	1279	15000	123	Sat Jun 22 07:41:23 2002
85	1278	15100	123	Sat Jun 22 00:40:45 2002
73	1277	15000	137	Fri Jun 21 14:17:05 2002
12	1276	15000	119	Wed Jun 19 21:09:28 2002
12	1275	60498	104 107 113 121	Wed Jun 19 21:05:52 2002
82	1274	78400	101 105 123 124 125	Wed Jun 19 21:05:00 2002
12	1273	15050	103	Wed Jun 19 21:03:34 2002
11	1272	15000	107	Wed Jun 19 20:40:09 2002
90	1271	15000	117	Wed Jun 19 19:51:30 2002
89	1270	15550	106	Wed Jun 19 18:13:04 2002
88	1269	60950	106 108 121 123	Wed Jun 19 15:42:15 2002
88	1268	60600	106 108 121 123	Wed Jun 19 15:41:40 2002
85	1267	15100	105	Wed Jun 19 09:46:51 2002
85	1266	45800	102 114 118	Wed Jun 19 09:30:27 2002
85	1265	48300	101 103 106	Wed Jun 19 09:27:50 2002
78	1264	44300	112 202 203	Tue Jun 18 18:52:52 2002
78	1263	15550	102	Tue Jun 18 18:48:35 2002
78	1262	45500	102 103 112	Tue Jun 18 18:47:27 2002
78	1261	29150	202 203	Tue Jun 18 18:46:02 2002
78	1260	43500	202 203 205	Tue Jun 18 18:40:16 2002
78	1259	58000	202 203 204 205	Tue Jun 18 18:36:16 2002
78	1258	30200	109 110	Tue Jun 18 18:31:53 2002

Gold

You are  
Provisional Winner

Yellow

Provisional Winner

White

Currently non  
Provisional Winner

# Illustration 7. Strategy Tools

**1. Select item(s)**

**2. Add selected to offer**

**3. Show as Winning**

**4. Return to [Provisional Winners Screen](#)**

**Your offer if submitted**

**This offer remains a Provisional Winner**

**Your offer bumps them**

**Offer was out and remains out**

**A partner used to bump packages with your offer**

**Query Results: 110 114 123 127**

run#	PIC	\$Offer	run#	PIC	\$Offer	run#	PIC	\$Offer	run#	PIC	\$Offer
101	76	17850	102	78	15550	103	12	15050	104	12	15150
105	85	15100	106	89	15550	107	12	15000	108	78	15150
109	76	15100	110	100	14950	111	100	14950	112	78	15000
113	12	15000	114	100	14950	115	100	14950	116	100	14950
117	90	15000	118	100	14950	119	71	15000	120	71	14950
121	12	15000	122	100	14950	123	12	15100	124	71	15000
125	100	14950	126	100	14950	127	100	15000	128	100	14950
129	100	14950	130	100	14950	131	100	14950	132	100	14950

PIC	bid#	price	items	time entered
10	1280	15000	110 114 123 127	Sat Jun 22 08:07:35 2002
85	1279	15000	127	Sat Jun 22 00:41:23 2002
85	1278	15100	123	Sat Jun 22 00:40:45 2002
13	1277	15000	137	Fri Jun 21 14:17:03 2002
12	1276	15000	119	Wed Jun 19 21:09:28 2002
12	1275	60450	104 107 113 121	Wed Jun 19 21:05:52 2002
12	1274	78400	101 105 123 124 125	Wed Jun 19 21:03:00 2002
12	1273	15050	103	Wed Jun 19 21:03:34 2002
41	1272	15000	107	Wed Jun 19 20:40:09 2002
90	1271	15000	117	Wed Jun 19 19:51:30 2002
89	1270	15550	106	Wed Jun 19 16:13:04 2002
88	1269	60950	106 108 121 123	Wed Jun 19 15:42:15 2002
88	1268	60600	106 108 121 123	Wed Jun 19 15:41:40 2002
84	1267	15000	105	Wed Jun 19 09:46:31 2002
84	1266	45000	107 114 118	Wed Jun 19 09:30:27 2002

## 6. Performance

Performance of the continuous combinatorial auction is addressed in three sections. The first section describes the experimental parameters. The section assumes an understanding of preference inducement, the nature of subject training and instruction. Much of what we know and can measure is derived from experiments and experimental testbeds. The second section is a sketch of the parameters in experiments and the third is a highlight of some experimental results. The third section reports the major properties of two field applications.

Experimental testbed methodology reflects an attempt to learn about mechanisms and environments that have never existed before in naturally occurring environments. Data from the field does not exist and appropriate data might even be impossible to get. Furthermore, the method operates in a world in which theory is suggestive but limited. The fact that the theory is incomplete suggests that “theory testing” is not necessarily a testbed objective because the answer to the question of whether the theory is true or not is already known. Certainly it is not true because it is incomplete and therefore vulnerable to a variety of sources of rejection.

Three questions are posed in a testbed. (1) Does the mechanism do what it is supposed to do? The question asks for a demonstration of proof of principle. (2) Does the mechanism do it does for understandable reasons? The question asks about a test of design consistency. The question asks if the result reflects the theory that was used in the design or are they simply random? Clearly, this is a basic question because it asks about the possibility that the design will scale. (3) Will the mechanism work in the proposed field environment? Of course, this third question is a key. It asks about the robustness of the theory when applied to possibly unknown conditions. It calls for tests under a variety of environments that could challenge the performance. The test environments might look nothing at all like “the real world” because the real world in which one might imagine the mechanism being deployed might not have conditions that theory suggests are stressful. On the other hand, testing in environments that might closely resemble the application environment might prove valuable in uncovering interactions with institutions and aspects of the environment that might not be anticipated by theory.. Institutional facts and environmental features can interact in surprising ways and have negative effects on performance. Examples of both types of environments are reported in the second section.

### Experimental Parameters

Standard experimental economics techniques are used to induce incentives. Of course, explaining preferences with synergies is a bit of a trick. Special techniques were used for that task.<sup>1</sup>

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<sup>1</sup> We will call our method of inducing preferences over sets “The basis method for synergies inducement”. A subject is given an array of subsets with a value attached to each that operates as a basis from which the value of any subset can be computed. The array is determined by the specific preference over sets, the pattern of preference synergies that the experimenter wants to induce. The array is a type of basis for computing the value of any subset. To find the value of an arbitrary set S start with the first element of the array and continue along the array to the very first subset of S, call it  $S_1$ . Record the value of  $S_1$ . Continue

Three classes of parameters existed for stress tests of the mechanism. The optimal allocations for the first sets are shown in Figures 1 and 3. Twenty items are to be allocated to five bidders. A representative indifference curve is drawn in the figure for each of the five bidders. Each of the bidders has an incentive to buy all twenty items should the prices be sufficiently low. The arrows are rough indicators of the gradient directions. The indifference curves cannot illustrate the synergies among the items, but the complementarities exist except when the items are “far” away from the maximum. For items near the maximum (about nine items), the purchase of any pair of items produces a value more than the sum of the values of the two items when the items are evaluated independently.

Efficiency is the measure used to assess performance. Assume that each individual bids the actual value of all packages. Under such assumptions the revenue that would be produced by the auction is a measure of potential “social benefits” and since the revenue is maximized it serves as a measure of the maximum possible benefits. In an experimental auction, the values are induced and are thus known to the experimenter. Thus, at the conclusion of the auction the allocations are known and the value of items to the bidder to whom they are allocated can be computed and summed. Call it the “total value received” by the bidders, independent of the prices paid.

auction efficiency = total value received/maximum possible surplus.

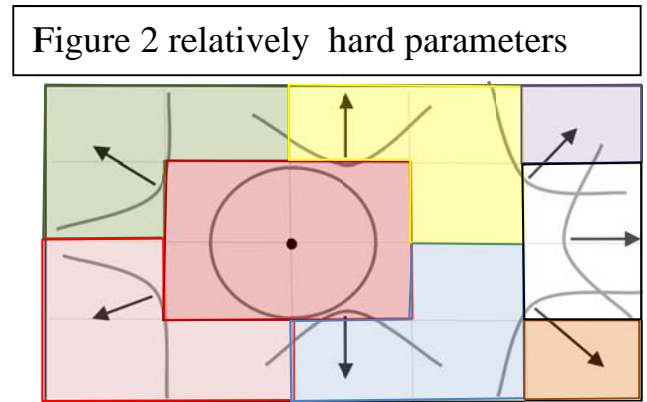
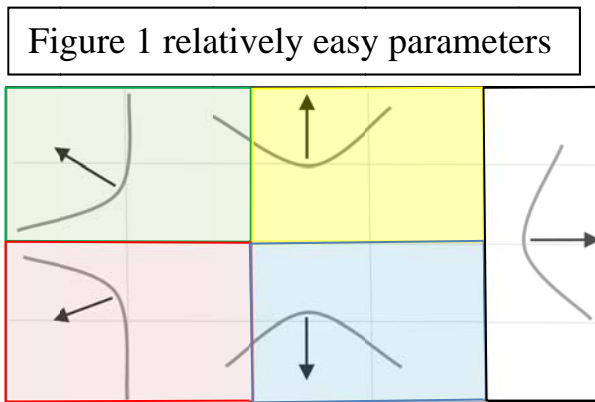
The efficiency is a type of cost/benefit measure only in the case of the auction in which the cost to the seller plays no role, there is no social cost, only benefits of the demand side. Typically, prices are an issue of income distribution as opposed to efficiency in allocation so the net benefits, value minus cost to bidders, are not part of the measurement.

The patterns of efficient allocations for the relatively easy parameters are Figure 1. As can be seen, four participants should acquire four adjacent items, resulting in four square patterns of allocations. The fifth bidder should acquire all items in the column to the right.

In the relatively hard parameters of Figure 2, four bidders have exactly the same preferences as existed in Figure 1. Three bidders are added and the resulting optimal pattern is illustrated in Figure 2. One of the new bidders should win the four units in the center. The two other new bidders should each acquire a unit at the extreme of the fifth column.

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along the array to the first subset of  $S \setminus S_1$ . Call it  $S_2$ . Record the value of  $S_2$ . Continue along the array to the first proper subset of  $S \setminus S_1 \setminus S_2$  and call it  $S_3$ . Record the value of  $S_3$ . Continue the process until all units of  $S$  have been included in a subset. Add the recorded values to get the value of  $S$ .



### Experimental tests

While many experiments were conducted, most lead to changes in the instructions and the interfaces and functionality of the mechanism. Nine small scale experiments and reported in Table 1. Five experiment in the table are based on the easy parameters and four use the hard parameters. In addition to the experiments in the table, the results of one reasonably large scale experiment are reported.

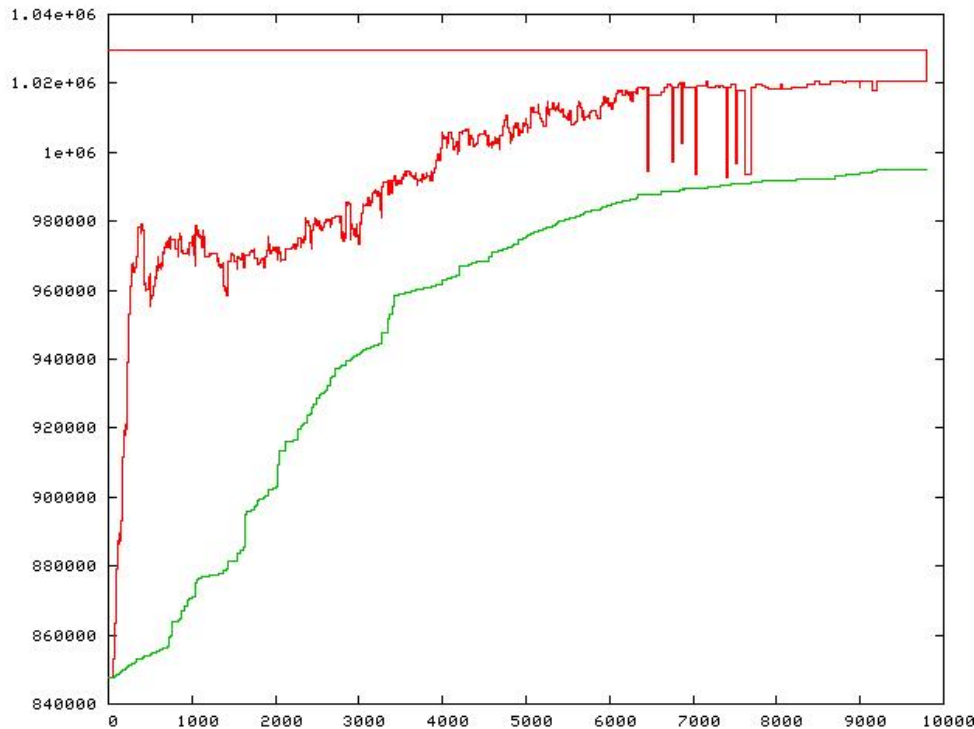
The five easy case parameter experiments were conducted near the end of the testing phase, when both software and instruction procedures had become stabilized. As can be seen, the experiments were regularly producing efficiencies near 100 %. Four hard parameter experiments are reported. The experiments on 060511 and the two on 060524 were with experienced subjects and resulted in an efficiency of 100%. The two experiments on 060525 were not conducted with appropriately trained subjects and are included as examples of what can be revealed by the testbed. The sources of the obvious subject misunderstandings were addressed following these experiments and were incorporated in modified training procedures as the project moved toward the applications.



**Table 1: Data**

DATA	Number of bidders	Efficient allocation value	Actual allocation value	Efficiency	note: experimental parameters
060411A	5	14296	14296	100%	easy
060411B	5	14296	14296	100%	easy
060517A	5	14926	14926	100%	easy
060519A	5	14296	14296	100%	easy
060519B	5	14296	14296	100%	easy
060511	8	10000	10000	100%	hard
060524	8	10000	10000	100%	hard
060525A	8	10000	8550	85.5%	hard – A person bought almost all – lost money
060525B	8	10000	6900	69%	hard – A key person bought only 1 and lost money on it.
					The two poor formers are included as examples that demonstrate the nature of the insights produced by testbed experiments.

**Figure 3.** Experiment 060526 50 items 30 bidders: Revenue and Efficiency.



As can be viewed in the figure, the revenue starts at a low level and rapidly increases, following an almost concave path and finally asymptotes at the level where the auction ends naturally as dictated by the clocks. The efficiency level converges to a level of approximately 90%. Revenues clearly approach an asymptote but as is the case with combinatorial auctions, there is some ambiguity about the appropriate equilibrium concept so the predicted revenue is not known.

#### Field applications

The results of two field applications are reported in this section. Of course, the details of parameters are unknown so efficiencies and maximum possible revenues are unknown. However, the time series are instructive and hold the impression of similarity to experiments conducted under laboratory conditions.

The results of an auction for 100 metric ton pallets of natural rubber are contained in Figure 4. Four internet bidders located around the world competed for 22 pallets of natural rubber located in a warehouse in Vietnam. The auction was conducted by the United Nations International Natural Rubber Organization, which had accumulated the pallets as part of a price stabilization program and was prepared to release the natural rubber back to private companies. Buyer identities were not public information.

The pallets were from different plantations. Natural rubber from a given plantation is a homogeneous product but rubber from different plantations has different and well known qualities. Starting bids were tendered by bidders as sealed bids. These bidders were accustomed to bidding in sealed bid environments and the initial bids are similar to other

sealed bids that the administrator for the INRO auction had observed and the bids were approximately market prices that exist in public markets. The initial bids on the rubber from different plantations reflected the difference of quality among the different plantations. Bids tended to be the same for rubber from the same plantation but bids differed for rubber from different plantations. Scale preferences were also evident. Bids for packages were frequently tendered. One bidder wanted all of the rubber in the warehouse and at the auction opening placed the high bid on all items for sale. This bidder ended the auction with ten pallets while the three other winners ended with 6, 5 and 1 pallets respectively.

Package bidding followed quickly after the initial bids. Some bidders expressed values for rubber from a limited set of plantations and others seemed to be interested in a mix with some sensitivity to price and quantity. The black dots appeared throughout the auction, signaling a bid on part of an existing larger package bid. The auction took about two hours and the “railroad lights” tended to appear signaling a threat to end the auction in the absence of bidding.

Total revenue in the INRO auction follows an approximately concave movement over time. If the starting revenue of \$884,975 is assumed to be the revenue that would have been produced by a sealed bid when compared with the \$927,000 auction revenue, the combinatorial auction produced about 5.5% more revenue. The flat places in the time series reveal instances of no bids (and thus the warning clock flashes) which increase in frequency as the auction progressed. Such patterns exist in experimental data.

The second field application examined is an auction for aquaculture sites located in Port Phillip Bay near Melbourne, Australia. The sites are appropriate for the growing of bivalve shellfish. The state of Victoria decided to auction eighteen sites. A total of ten bidders participated and bid for 18 sites. Seven bidders were winners producing \$575,000 in revenue. The sites were scattered across six locations. Bidders were interested in scale since they must meet regular demand for deliveries. They are also interested in a portfolio of sites reflecting a diversity of location due to currents, winds, possible diseases, and location relative to home base and delivery points. Thus multiple synergies existed and package bids were used frequently<sup>2</sup>.

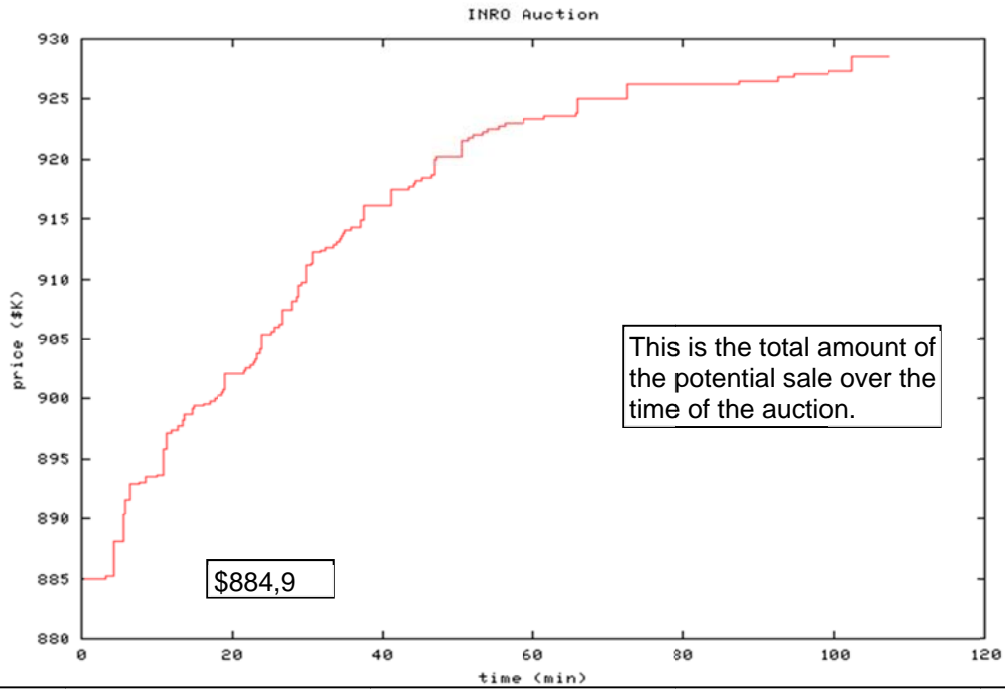
The revenue and timing from the aquaculture auction are displayed in Figures 5a, 5b and 5c. Figure 5a demonstrates the typical concavity of revenue when displayed in clock time over the approximate 2 hours of the auction. The delay in the middle of the figure reflects an equipment problem that delayed the auction for about five minutes<sup>3</sup>. Bids are entered rapidly at first (shown in Figure 5b) and then slow down as the auction advances

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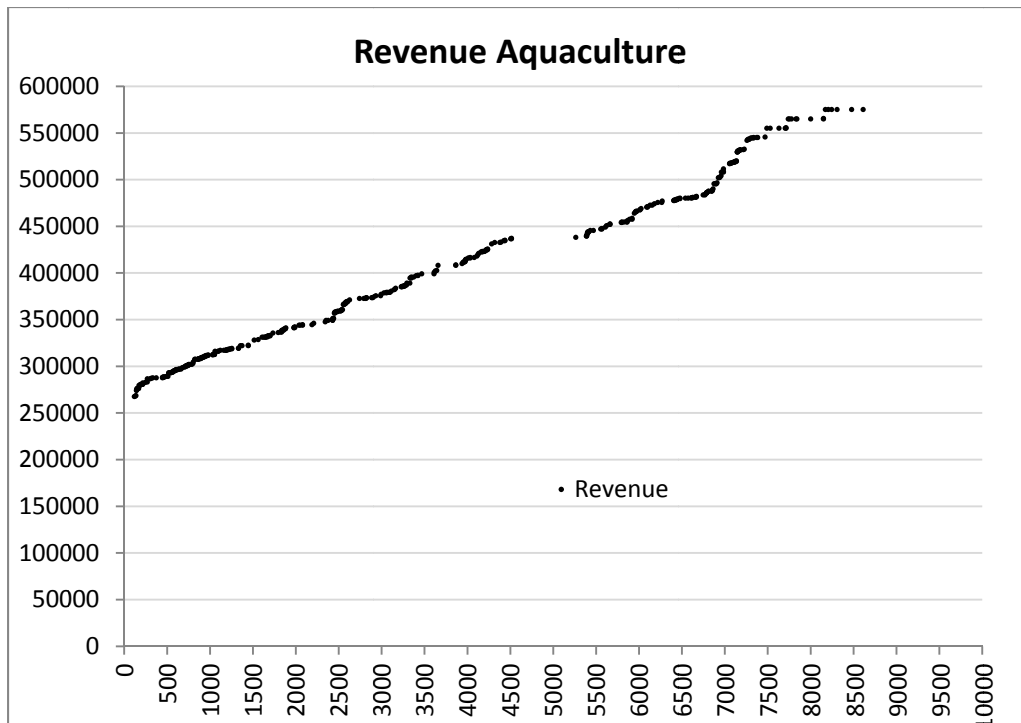
<sup>2</sup> The total number of bids was 300 of which 129 were bids for packages of items and 171 were bids on single items. The auction lasted about 7000 seconds which means that a bid or ask was arriving every 5 seconds or so. There were 1032 query, about 3.5 query per bid, so something was happening about every 4 seconds not counting cancellations and other activities.

<sup>3</sup> Someone unplugged to power to the server at the remote location where the auction was held. The event illustrates the need for review and testing of every mode in which an online auction might fail. The fact that the recovery was complete and fast, with no apparent disruption to the auction suggests the existence of considerable background research not covered in this brief paper.

and become very slow at the end (shown in Figure 5b). This pattern is very reminiscent of the behavior of continuous auctions



**Figure 4.** International Natural Rubber Organization 4 bidders 22 items (100 metric tons of natural



**Figure 5a:** Aquaculture Revenue and Timing of all Bids.

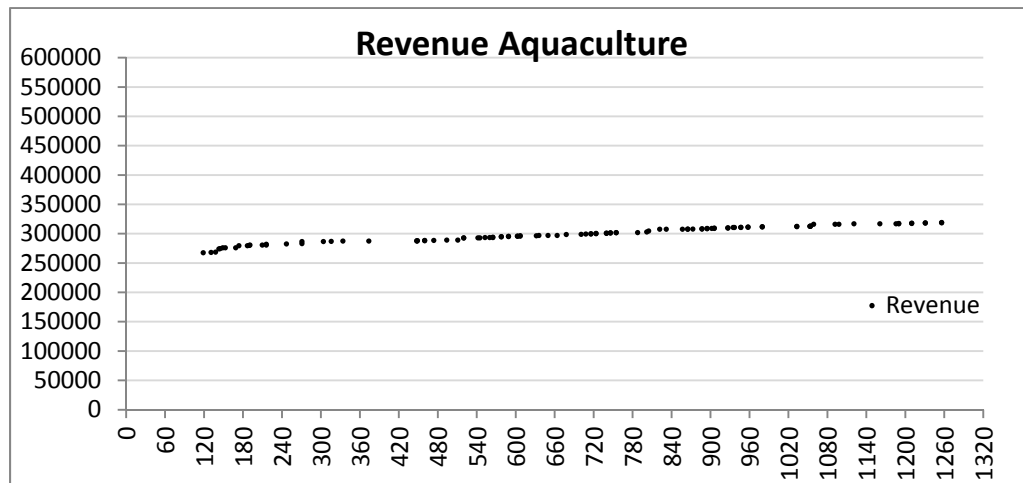


Figure 5b: Timing of Bids (first 20 minutes).

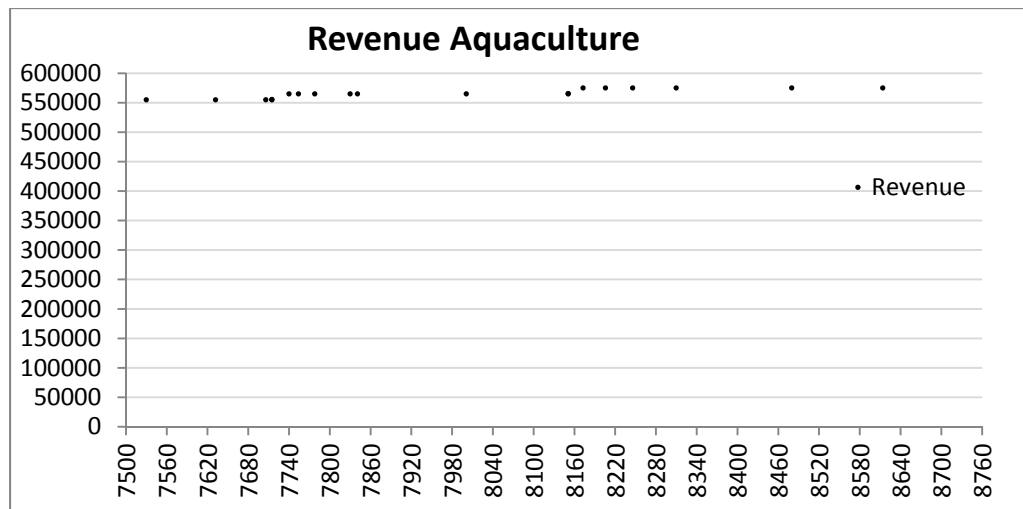


Figure 5c: Aquaculture Timing of Bids (last 20 minutes)

## 7. Summary and Observations

Several features of the mechanism are worth emphasis. The absence of a concept of a price per item is a departure from tradition. Replacing the measures contained in prices are queries and displays that can respond to human pattern recognition and crafted information needs. Obviously the economic content of a concept of prices is working in the background but the operation of the mechanism is not based on their use. The use of clocks is important. They carry key public information and create the proper level of incentives for coordination. A bidder need only meet an increment requirement to keep a

negotiation alive before facing an all or none choice of implementing a “contribution” to the public good of breaking up a large bid or collections of bids and becoming a provisional winner. The dots provide feedback by calling attention to actions of others and the possible intentions that underlay the actions of others play. This type of information that contributes to coalition formation plays a key role.

Computational problems can clearly pose problems as the size of the auction grows but the ability to solve big problems depends on the structure of the problem and the computing technology. Continuous combinatorial auctions much larger than reported here have been conducted. The computations times we encountered were all measured in fractions of seconds even when hundreds of bids exist in the set of all bids. Of course, ways exist to reduce the computational problem at the expense of limitations on permissible bids. Such restrictions have been tested but not used.

The testbed methods have some departures from what an untutored theorist might expect. The methods are designed to address problems where the theory is not complete and might be no more than suggestive. Tests of such theories do not make much sense when research is confronted by a scale of limited budget, limited time, and an unbounded infinity of variables. Yet, the role of theory plays a fundamental role. Theory, regardless of how incomplete it might be, is the tool that takes the analysis from the limited observations under controlled conditions to the substantially unknown conditions of the field. The theory must be robust and the testbeds help establish that.

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